U.S. DEPARTMENT OF ENERGY

PNNL- 18671

Prepared for the U.S. Department of Energy
Under Contract DE-AC05-76RL01830

Topic Brief 2: Sample Size

Updated 12/1/2009

Pacific Northwest
NATIONAL LABORATORY

The American Recovery and Reinvestment Act (Recovery Act) requires the governors to notify the Secretary of Energy that the governor has obtained assurances that the state or applicable units of local government will implement a plan for achieving compliance with the state or local government building energy code(s) in at least 90% of the new and renovated residential and commercial building space. A definition of how this compliance should be measured is not provided, but DOE's Building Energy Codes Program (BECP) recommends this measurement be based on evaluation of a statistically significant number of buildings within the state. Each building will be assigned a compliance rate based on the proportion of weighted code requirements that each has passed. The evaluated buildings' scores within a state are averaged to derive an overall compliance metric with an associated confidence. This topic brief describes the methodology used to derive the recommended sample size in each state. The results include a recommended minimum sample size of 44 residential buildings, and 44 commercial buildings distributed among three main building size strata[1]. Renovations will be treated as separate populations, and will be described in a separate topic brief.

Stratification and Population Definition

The BECP recommends that new commercial buildings and residential buildings be treated as two separate populations, each of which will be evaluated and reported separately. The required sample sizes described below must be generated for both populations.

For each population, the 44 samples will be distributed across the climate zones in that state, with the number of samples in each climate zone determined by the most current building start information available for each climate zone. The samples distributed in each climate zone will then be distributed across the most active jurisdictions, again based on building start information. This stratified randomization will help provide a balanced set of information without losing the advantages of random sampling. Commercial buildings will be further stratified according to building size.

There are unique attributes of buildings which could be influential in biasing the compliance estimates for a state (e.g., differences in building size, use, location, builder/contractor, etc.). The BECP guidelines will identify important attributes that, if possible, should be represented in the sample of buildings from each state.

A separate topic brief, *Topic Brief 5, Sample Distribution and Make-Up*, provides more details about how the samples are distributed across climate zones and jurisdictions, and the building attributes (e.g., size and use) that are recommended to be included.

Sample Size

The BECP recommends that state sample sizes be calculated using Equation 1. This equation can be used to identify an appropriate sample size for a one-sided confidence interval. Typical two-sided confidence intervals give the mean of the sample and then provide a margin of error above and below that sample mean. This margin of error describes where the true population mean could exist with a prescribed confidence (e.g., the proportion of the population voting for candidate A is 52% ± 3% with 95% confidence). One-sided confidence statements are only interested in one side of the range; in this case we are interested in the higher range. The two-sided equation is very similar to the one-sided equation, but requires a slightly larger sample size.

[1] Very large commercial buildings will be added to the commercial building sample size in many states. Refer to *Topic Brief 5*, *Sample Distribution and Make-Up*, for the size strata delineations.

$$n = \frac{S^2\left(Z_{(1-\alpha)} + Z_{(1-\beta)}\right)^2}{\Delta^2} + 0.5Z^2_{(1-\alpha)} \quad (1)$$

where

- n = the number of buildings that must be evaluated from the state
- s^2 = the square of the standard deviation (sample variance)
- Z = a standard normal score from a normal distribution
- $1-\alpha$ = the confidence level
- $1-\beta$ = the power
- Δ (delta) = the minimum true difference from 90% that is important to correctly detect as being different from 90% (i.e., the detectable difference)

Parameters Assumed for Calculations

Typically an $\alpha = 0.05$ and $\beta = 0.2$ are used and have been assumed for this effort. The BECP has also identified the value for Δ as 5%. The average state building compliance rate can be between 0 and 100% and have standard deviation estimates as large as 50%. To identify an appropriate standard deviation estimate, *s*, for this study, the BECP evaluated a range of different scenarios when a state's building compliance score is close to 90%. Based on this evaluation, an estimated standard deviation of 13 percent was selected and is believed to be a conservative estimate (note that the variance is $s^2=13^2$).

Final Statistical Statements and Data Analysis

The final statistical confidence statements will provide each state with the ability to say, "The state of XXX is 95% confident that the upper confidence bound of the building compliance rate is YY% + ZZ%." If the resulting upper confidence bound is above 90% we conclude that the state is meeting the Recovery Act legislation, that is, the data gives some evidence (with prescribed confidence level) that the state is in compliance.

For example, State A and B went through the entire process of a random sampling (n=44 for this example), data collection, and data summarization. State A had an average building code compliance rate of 82% and a standard deviation estimate of 12% and State B had an average building code compliance of 87.5% and an estimated standard deviation of 15%. Using Equation 1, with a confidence level = 95%, the upper confidence bound is calculated using the following equation,

$$\bar{x} + 1.645 \times \frac{s}{\sqrt{n}} \quad (2)$$

where

- $\bar{x}$ = the mean
- s = the standard deviation
- n = 44 (the number of buildings in the sample)

State A would report an upper confidence bound of their compliance rate as 84.98%:

$$82\% + 1.645 \times \frac{12\%}{\sqrt{44}} = 84.98\%.$$

State B would report an upper confidence bound of their compliance rate as 91.22%:

$$87.5\% + 1.645 \times \frac{15\%}{\sqrt{44}} = 91.22\%$$

State A's upper confidence bound provides some evidence that their compliance rate is below 90% with 95% confidence, and would therefore not meet the 90% compliance target. State B's upper confidence bound of 91.22% provides some evidence that they would meet the 90% compliance goal. Any state that has a compliance rate over 90% from an appropriate sample of buildings would be deemed to meet the Recovery Act goal of 90% compliance.

The final confidence interval is based on the calculated standard deviation from the actual state sample and not the assumed standard deviation used in defining the sample size (which was assumed to be 13% in the previous section). We have tried to make conservative assumptions about the standard deviation observed in the compliance rates from the sample of buildings and expect most standard deviations to be smaller.

It is understood that the standard deviation from a state's sample of buildings is an estimate and some allowance will be made for this fact. Thus, the result of a state's sample standard deviation can be as high as .16 and be acceptable. If the standard deviation estimate resulting from the state sample is above .16 and the upper confidence bound is above .9, then additional samples based on the estimated standard deviation will be necessary to ensure a valid sample. The number of additional samples, which can be determined from Equation 1.1 using the state's estimated *s,* will then be incorporated into the state's building compliance upper confidence calculation.

Figure 1 shows the different sample size requirements as a function of the assumed standard deviation estimates. The BECP assumed sample size is labeled on the graph.

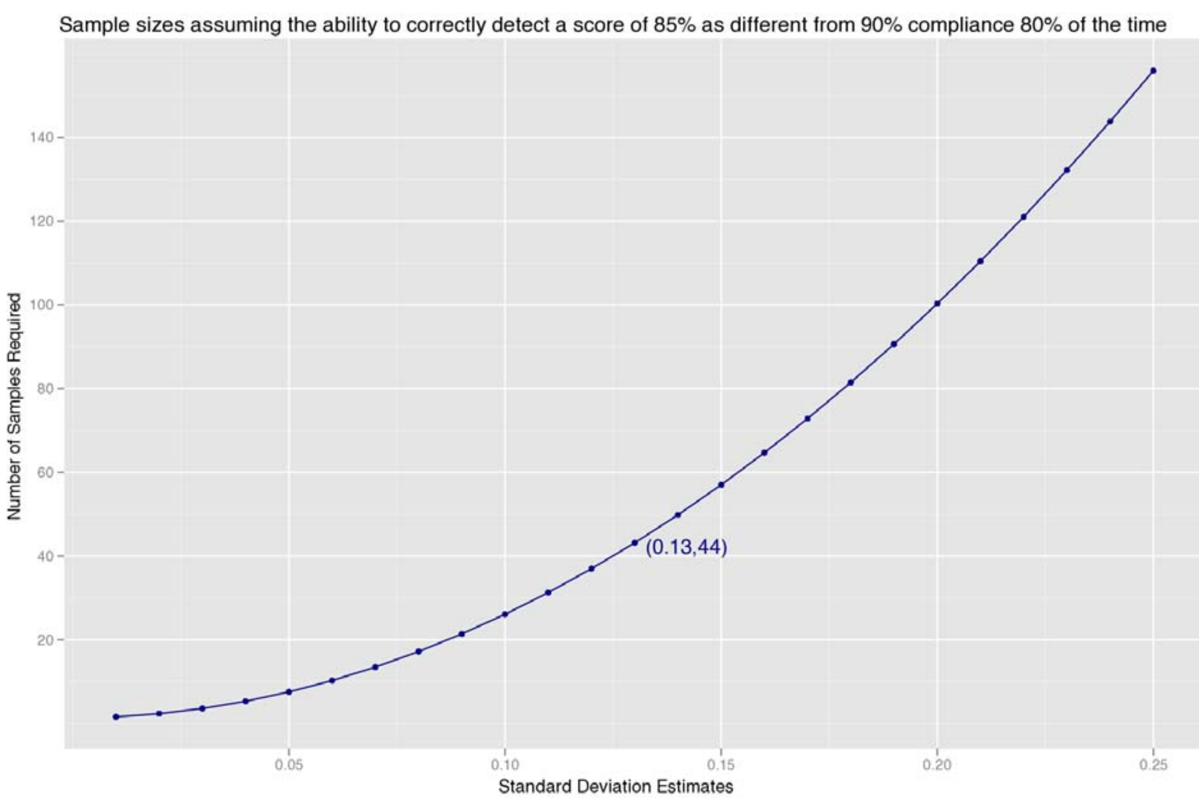

Figure 1: Graph of the required sample sizes based on different standard deviation estimates. The BECP assumed standard deviation is labeled on the graph.

Small Population Sample Size Adjustment

The sample size calculations proposed above are based on the assumption that the population of new buildings within a state during a study period is large (i.e., over 1000 buildings). If the population of buildings for a state's study period is smaller, the required sample size can be reduced. This reduction is of the amount shown in the following equation:

$$n_{adjusted} = \frac{n}{1 + \left(n / N \right)}$$

where

$n_{adjusted}$	=	the adjusted number of required samples, rounded up to the nearest integer
n	=	the original sample size estimated, which the BECP has recommended to be 44
N	=	the total number of buildings expected to be in the study period population

For example, the state of North Dakota does not have a large population of commercial buildings to study in any given year. If the state of North Dakota was going to evaluate its building performance over the upcoming two-year period, it could use the total number of buildings built in the previous two years as an estimate of the expected population (114 buildings in 2006 and 97 building in 2007). The total of 221 buildings can be used for an estimate of *N* in the above equation. Thus, 37 observations would be the minimum sample required for the state of North Dakota.

$$n_{adjusted} = \frac{44}{1 + \left(44 / 221 \right)} = 37$$

References

1. Gilbert, R.O. 1987. *Statistical Methods for Environmental Pollution Monitoring*. Van Nostrand Reinhold, New York. (Same text available as Gilbert, RO. 1997. John Wiley & Sons, New York.)